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ON THE PROBABILISTIC ORIGIN OF DIAL'S MULTIPATH TRAFFIC ASSIGNMENT MODEL

by

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ABSTRACT

Dial's multipath traffic assignment model is interpreted as a probabilistic choice model. Anomalies in path assignments generated by Dial's algorithm are related to the independence-of-irrelevant alternatives axiom.

Introduction

Dial's (1971) multipath traffic assignment model has stimulated great interest among researchers and practitioners of transportation planning. The ingenious algorithm developed by Dial offered immediate advantages over the "all-or-nothing" traffic assignment method in that it apportioned trips to alternative reasonable paths in a way which is related to the length (cost) of these paths. In addition, the procedure required a modest additional computational effort when compared to the "all-or-nothing" method.

An attractive feature of the multipath assignment method is that, between an origin and a destination, as paths get longer they become less likely to be used and the user of the model has the facility to calibrate the algorithm to suit his needs. In particular, consider an origin-destination pair (u,v) and the set of paths P that are admissible between them. The path flows are h_p , $p \in P$.

Dial's clever mechanism achieves a subdivision of the total demand, d_{uv} , for the pair (u,v) in the following proportions

$$\text{Prob}\ (p) \;=\; K \,.\, \exp^{\theta(t^{*}-t_p)} \;, \qquad (1)$$

where t^{*} is the length of the shortest path between (u,v) and t_p is the length of path p . The flow on path p is

$$h_p \;=\; d_{uv} \,.\, \text{Prob}\ (p) \qquad (2)$$

Though not crucial to the main thrust of this paper, we now interpret "Prob (p)" for logical completeness; Dial's paper appears to leave the misleading impression that this is a genuine "probability".

Strictly speaking, "Prob (p)" is a misnomer. It is not a probability. Rather, it is a proportion: the fraction of (u,v) trips that use p. Thus, Dial's algorithm produces a flow on p that is deterministic, not random. More specifically, h_p is always the actual flow on p, not merely its expected value. In its original presentation Dial's model is an aggregate model. It is important to point out that, when arc lengths and origin-destination demands are deterministic, the only way to get truly probabilistic behavior is via behavioral models that postulate probabilistic individual choice mechanism.

Since h_p increases proportionally to d_{uv} the algorithm is sometimes referred to as a "proportional method". The user may calibrate the parameter θ.

Since we have that over all the admissible paths between (u,v)

$$\sum_{p \in P} \text{Prob}\ (p) = 1 , \tag{3}$$

it is easy to show that

$$\text{Prob}\ (p) = \frac{\exp^{-\theta t_p}}{\sum_{p \in P} \exp^{-\theta t_p}} . \tag{4}$$

It is this form of the "probability" of using a path that we will show to be most revealing of the probabilistic origin of this method.

Dial's paper stimulated Robillard (1974a, 1974b) to generalize the method to consider dynamic inputs. Trahan (1974) refined the computational steps of algorithm. Florian (1974) suggested an indirect way of introducing congestion effects. The algorithm, in its original form, was programmed and is available as a subroutine, entitled STOCH, that was used in numerous applications, including an interactive public transit planning method developed by Rapp et al (1974).

However, users of Dial's algorithm noted that the resulting flows often have some seemingly undesirable features. In particular Schneider (1973) and then Burrell (1974), among others, noted that the example given in Figure 1 leads to unreasonable flows.

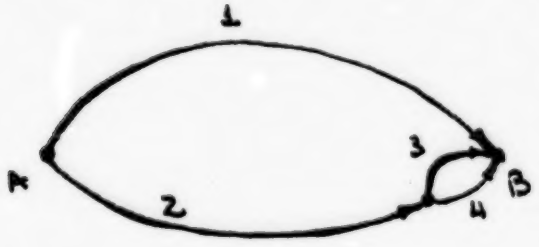

Figure 1

All the routes from A to B are of equal length. The probabilistic assignment method would assign 1/3 of the trips to link 1, 2/3 of the trips to link 2 and 1/3 of the trips to links 3 and 4. This is counter-intuitive since links 3 and 4 were chosen to be of very small length compared to link 2 and we would expect links 1 and 2 to carry comparable flows.

Schneider (1973) attempted to explain this defect by deriving the flow distribution rule (4) from the unreasonable behavioral assumption that

> "as travellers come to the end of a link in the network they choose, at an intersection, the next link in a completely random fashion, subject only to network connectivity conditions and to a total travel time constraint".

While we could not follow Schneider's derivation in detail, we are of the opinion that even if (4) may be derived from this implausible behavioral assumption, the peculiarities of the method are still not properly explained. Schneider did not claim that the reverse implication holds, namely that (4) implies the behavioral assumptions. This leaves open the possibility (which we consider unlikely) that (4) may be consistent with more plausible behavioral assumptions. We intend to give a thorough explanation of the defects pointed out above by simply recognizing (4) as a multinomial logit model and studying the consequences.

Choice among paths

Consider the traffic assignment problem in the following individual (disaggregate) context. A traveller from origin u to destination v faces the paths $p \in P$, which is each described by its length t_p. For the moment, let us regard "Prob (p)" as a bona fide probability. We then interpret h_p as the expected flow on p, giving a measure of aggregate behavior of all travellers from u to v. With this

somewhat strained interpretation, the choice among paths is given by a multinomial logit model, whose origin is probabilistic choice theory developed by Luce (1959) and described in the context of transportation demand models by Charles River Associates (1972).

As is well known (see for instance Brand and Manheim (1973)), the multinomial logit model implies a choice mechanism known as the independence-of-irrelevant-alternatives axiom. Briefly, this axiom states that the ratio of the probabilities of choosing one alternative to that of choosing another is independent of the probability of choosing other available alternatives. It is easy to verify that (4) satisfies the axiom by considering paths p_1 and p_2. The ratio of their probabilities of choice is

$$\text{Prob}\ (p_1)\ /\ \text{Prob}\ (p_2) = \frac{\exp^{-\theta t_1}}{\exp^{-\theta t_2}} \qquad (5)$$

and does not depend on the number of admissible paths between u and v. In fact (4) implies the axiom, but not conversely. To see that the converse does not hold, one could choose each alternative with equal probability, independently of path length. We would like to show next that the choice among paths in Dial's method, which implies the axiom of the independence-of-irrelevant-alternative, combined with his definition of a path, is the reason for the undesirable feature exhibited by the example in Figure 1.

Consider the example in Figure 2

Figure 2

Two routes of equal length connect A and B. The probabilities that would be assigned to path {1} and {2,3} are equal, namely $\frac{1}{2}$, and their ratios is 1. Suppose now that link 4 is added and the new network is now given by Figure 1. The traveller from A to B now chooses between 3 paths of equal length: {1}, {2,3} and {2,4}. (A key feature of Dial's method is that it ignores the fact that paths {2,3} and {2,4} differ only in a minor way.) Since these paths are all of equal length, and since the above-mentioned axiom is satisfied, each of these paths is chosen now with a probability of $\frac{1}{3}$ and, as a consequence, link 2 receives double the traffic of link 1. The reason this situation is implausible is that we expect the traveller to perceive {2,3} and {2,4} together as a single alternative against {1} and not treat them as "independent" alternatives. The application of choice models that satisfy this axiom is most successful when the alternatives are perceived as being independent by the choice maker. Otherwise, the independence axiom may be too strong an assumption to make. A detailed analysis of the weakness of this axiom is given by Luce (1959, pp. 131-134) and Charles River Associates (1972, pp. 5.25-5.26).

With Dial's definition of a path, distinct paths often have some common links; intuitively, such paths (alternatives) are not independent. A more stringent definition of independence would even prohibit common nodes, except for origin and destination. Thus, we conclude that the main drawback of the probabilistic assignment method is that its choice mechanism satisfies the above axiom of choice and, due to the implicit definition of paths used, the alternative paths in the admissible set of paths P are not independent, except for special network structures.

Conclusions

The above discussion has some implications regarding the use of the probabilistic assignment method in practice. It is clear that it is too difficult to ensure, in any given application, that unreasonable flows, such as the example of Figure 1, do not occur. Thus, it seems to us that for applications in urban areas, where congestion effects are predominant, it is desirable to use an assignment method which explicitly considers congestion effects, such as Nguyen (1974). Fox (1975) gives a discussion of such a method from a probabilistic viewpoint. If congestion effects are truly minor and the arc lengths and origin-destination demands are constants, any reasonable assignment method will produce an assignment that is close to "all-or-nothing". If Dial's algorithm is so calibrated (θ large), its pathologies ordinarily would not be apparent and, in this case, probably not important, unless there are many paths of identical shortest length that are not link independent.

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